

(i) *Conditions at the elastic wavefront*

In general, material constitutive relations postulate a limiting magnitude for the deviatoric stress below which plastic strain does not occur and we denote this magnitude by  $2Y/3$ . In some constitutive relations,  $Y$  can have the value zero. For steady-state conditions the deviatoric stress in the elastic wavefront will be relaxed to (minus) this value (since the wave is compressive):

$$(\rho_0/\rho_h)(\sigma_h + p_h) = -(2Y/3). \quad (31)$$

Comparing (21) and (31) shows that

$$Y/2G = \ln(\rho_h/\rho_0). \quad (32)$$

From (10) and (22),

$$p_h = (\rho_h/\rho_0)K \ln(\rho_h/\rho_0) + (\rho_h/\rho_0)K\beta T_0[(\rho_h/\rho_0)^\mu - 1]. \quad (33)$$

In view of (32), subtracting (33) from (31) gives

$$\sigma_h/K = -(2Y/3K)(1 + 3K/4G)(\rho_h/\rho_0) - (\rho_h/\rho_0)\beta T_0[(\rho_h/\rho_0)^\mu - 1]. \quad (34)$$

Finally, we note that the temperature  $T_h$  at the wavefront is given by (22). Thus, all of the variables at the wavefront are parametrically expressible in terms of the wavefront density  $\rho_h$  which is given as a function of  $Y$  by (32).

(ii) *Conditions past the plastic wavefront*

Infinitely far behind the steady-state shock the material will be compressed to its maximum density  $\rho_m$ . Inspection of (20) and (10) shows that the maximum temperature,  $T_m$ , and maximum pressure,  $p_m$ , occur here as well. However, in order to calculate their values it is necessary to also consider the corresponding axial stress.

As at the precursor wavefront infinitely far ahead of the plastic wave, the material infinitely far behind the plastic wave will be fully relaxed, that is, the deviatoric stress will satisfy a condition such as

$$(\rho_0/\rho_m)(\sigma_m + p_m) = -2Y/3. \quad (35)$$

Here,  $\sigma_m$  is the axial stress corresponding to  $\rho_m$ . From (23) it is clear that this is the algebraically largest stress that occurs (but since its sign is negative, we hesitate to use the term maximum). Equation (35) is simply a direct extension of the yield criterion of (31) which postulates that the shearing strength of the material is dependent upon the degree of compression. Alternatively, these two equations could be written having simply  $(\sigma + p)$  equal to a constant. At present very little experimental evidence exists relating to this point but work by BROBERG (1956) and CURRAN (1963) seems to indicate that shear strength is more nearly compression-dependent than constant.

In most materials there is a strong dependence of yield strength upon temperature. Such dependence can be readily included in the computations since an iterative procedure is already required as explained below. However, in the examples given here, the yield strength was simply taken to be constant.

For a given  $\rho_m$ , equations (26), (10), (23) and (35) must be solved by successive approximations to find a consistent set of values for  $T_m$ ,  $p_m$ ,  $\sigma_m$ ,  $\rho_0 V_0^2$  (and  $2Y/3$  if this quantity is taken to be temperature dependent).

(iii) *Intermediate conditions*

Most other variables of interest will take on their largest values at some density intermediate between  $\rho_h$  and  $\rho_m$ , and usually within the actual shock front. Consider,

for example, the shear stress  $3(\sigma+p)/4$ . By adding (10) and (23) we obtain

$$(\sigma+p) = \rho_0 V_0^2 + \sigma_h + K \ln(\rho/\rho_0) + K\beta(T-T_0), \quad (36)$$

The largest value of shear stress can easily be found for a given wave speed. Some elementary numerical technique such as interval-halving is applied to the equation  $d(\sigma+p)/d\rho = 0$  to find the density,  $\rho_1$ , at which the derivative is zero. The corresponding value for  $(\sigma+p)$  is then obtained from (36), using (26), with  $\rho$  set equal to  $\rho_1$ .

A similar technique is applicable to the plastic strain rate although here it is more convenient to use the chain rule for differentiation. Denote the variables determining  $\dot{\epsilon}^p$  by  $x_i$  so that

$$\dot{\epsilon}^p = f(x_1, x_2, \dots). \quad (37)$$

Then,

$$\frac{d\dot{\epsilon}^p}{d\rho} = \frac{\partial f}{\partial x_i} \cdot \frac{dx_i}{d\rho} \quad (\text{summed over } i), \quad (38)$$

where the factors  $dx_i/d\rho$  can be determined from equations such as (17), (23), (26), etc.

Among the other variables that can be treated in this manner, the total strain rate and rate-of-change of density both come out quite like the plastic strain rate. The algebra for these computations was developed in (KG) and can easily be extended to include temperature as a constitutive parameter, as indicated by (37) and (38).

Finally we note that shock thickness was defined in (KG) through a characteristic time equal to the total change of density,  $\rho_m - \rho_h$ , divided by the maximum rate of change of density. This is the shock thickness in the time coordinate and is converted to a spatial distance by multiplying by the wave speed  $V_0$ . The ratio of this thickness,  $\delta$ , to a reference distance  $\Delta = \tau(K/\rho_0)^{1/2}$  is a convenient non-dimensional measure of shock thickness. Here,  $\tau$  is a characteristic relaxation time which occurs in (37) and  $(K/\rho_0)^{1/2}$  is the hydrodynamic wave speed.

## 5. EXAMPLES

To illustrate the foregoing numerical method we consider two material constitutive equations. The first is

$$\dot{\epsilon}^p = -(1/\tau)(1 - M\dot{\epsilon}^p) [-\rho_0(\sigma+p)/\rho S]^n. \quad (39)$$

Here,  $\tau$  is a characteristic relaxation time, and  $M$ ,  $S$  and  $n$  are other material constants. Equation (39) gives the plastic strain rate as independent of temperature and is treated only because it was considered in (KG) and therefore provides an opportunity to examine the temperature effect introduced only through the pressure-density relation. As before, we take  $G = 3K/8$ ,  $M = 10^3$ ,  $S = 10^{-3}$  K and  $n = 10$ . The material described by (38) is analogous to a Maxwell material in that  $\dot{\epsilon}^p = 0$  only for  $\sigma+p = 0$ . Thus, the appropriate value for  $Y$  is 0, and correspondingly  $\rho_h = \rho_0$ ,  $\sigma_h = 0$  and  $T_h = T_0$ . For the present calculations we also take  $\beta T_0 = 10^{-2}$  and  $\mu = 5$ . The result of (KG) can be retrieved by setting  $\beta = 0$  which for brevity will be referred to as the *non-thermal case*.

Figures 1 and 2 indicate the variation of temperature with density for the material parameters cited above. In Fig. 1, the ratio of maximum to initial temperature is